INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MARKSCHEME

May 2001

MATHEMATICAL METHODS

Standard Level

Paper 1

11 pages

1. (a) Median = middle number of 75 ***(M1)***

= 38th number

= 4 ***(A1)*** ***(C2)***

(b) Mean $= \dfrac{5+18+48+72+100+42}{75}$ ***(M1)***

$= \dfrac{285}{75}$

$= 3.8$ ***(A1)*** ***(C2)***

[4 marks]

2. $y = (x+2)(x-3)$ ***(M1)***

$= x^2 - x - 6$ ***(A1)***

Therefore, $0 = 4 - 2p + q$ ***(A1)(A1)*** ***(C2)(C2)***

OR

$y = x^2 - x - 6$ ***(C3)***

OR

$0 = 4 - 2p + q$ ***(A1)***

$0 = 9 + 3p + q$ ***(A1)***

$p = -1, q = -6$ ***(A1)(A1)*** ***(C2)(C2)***

[4 marks]

3. (a) $\dfrac{15.2}{1.027} = 14.8$ million ***(M1)(A1)*** ***(C2)***

(b) $\dfrac{15.2}{(1.027)^5} = 13.3$ million ***(M1)(A1)*** ***(C2)***

OR

$\dfrac{14.8}{(1.027)^4} = 13.3$ million ***(M1)(A1)*** ***(C2)***

[4 marks]

4. (a) The smallest angle is opposite the smallest side.

$\cos\theta = \dfrac{8^2 + 7^2 - 5^2}{2 \times 8 \times 7}$ ***(M1)***

$= \dfrac{88}{112} = \dfrac{11}{14} = 0.7857$

Therefore, $\theta = 38.2^\circ$ ***(A1)*** ***(C2)***

(b) Area $= \dfrac{1}{2} \times 8 \times 7 \times \sin 38.2^\circ$ ***(M1)***

$= 17.3 \text{ cm}^2$ ***(A1)*** ***(C2)***

[4 marks]

5. $y = \sin(2x-1)$

$\dfrac{dy}{dx} = 2\cos(2x-1)$ ***(A1)(A1)***

At $\left(\dfrac{1}{2}, 0\right)$, the gradient of the tangent $= 2\cos 0$ ***(A1)***

$= 2$ ***(A1)*** ***(C4)***

[4 marks]

6. $(3x+2y)^4 = (3x)^4 + \binom{4}{1}(3x)^3(2y) + \binom{4}{2}(3x)^2(2y)^2 + \binom{4}{3}(3x)(2y)^3 + (2y)^4$ ***(A1)***

$= 81x^4 + 216x^3y + \mathbf{216x^2y^2 + 96xy^3 + 16y^4}$ ***(A1)(A1)(A1)*** ***(C4)***

[4 marks]

7. P(different colours) $= 1 - [\text{P(GG)} + \text{P(RR)} + \text{P(WW)}]$ ***(M1)***

$= 1 - \left(\dfrac{10}{26}\times\dfrac{9}{25} + \dfrac{10}{26}\times\dfrac{9}{25} + \dfrac{6}{26}\times\dfrac{5}{25}\right)$ ***(A1)***

$= 1 - \left(\dfrac{210}{650}\right)$ ***(A1)***

$= \dfrac{44}{65} (= 0.677, \text{ to 3 s.f.})$ ***(A1)*** ***(C4)***

OR

P(different colours) $= \text{P(GR)} + \text{P(RG)} + \text{P(GW)} + \text{P(WG)} + \text{P(RW)} + \text{P(WR)}$ ***(A1)***

$= 4\left(\dfrac{10}{26}\times\dfrac{6}{25}\right) + 2\left(\dfrac{10}{26}\times\dfrac{10}{25}\right)$ ***(A1)(A1)***

$= \dfrac{44}{65} (= 0.677, \text{ to 3 s.f.})$ ***(A1)*** ***(C4)***

[4 marks]

8. Gradient of PQ $= \dfrac{7-0}{-5-4} = -\dfrac{7}{9}$ ***(A1)***

Gradient of perpendicular line $= \dfrac{9}{7}$ ***(M1)***

Required equation: $y - 0 = \dfrac{9}{7}(x-4)$ ***(A1)***

$7y = 9x - 36$

$9x - 7y - 36 = 0$ ***(A1)*** ***(C4)***

OR

$\begin{pmatrix} 9 \\ -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ -7 \end{pmatrix}\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ ***(M1)(A1)(A1)***

$9x - 7y - 36 = 0$ ***(A1)*** ***(C4)***

[4 marks]

9. **Note:** Do not penalize for the omission of C.

(a) $\int \sin(3x+7)\,dx = -\frac{1}{3}\cos(3x+7) + C$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $\frac{1}{3}$, ***(A1)*** for $-\cos(3x+7)$.

(b) $\int e^{-4x}dx = -\frac{1}{4}e^{-4x} + C$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $-\frac{1}{4}$, ***(A1)*** for e^{-4x}.

[4 marks]

10. $\cos\theta = \frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$ ***(M1)***

$= \frac{-4+14}{\sqrt{20}\sqrt{50}}$ ***(A1)***

$= \frac{10}{10\sqrt{10}}$

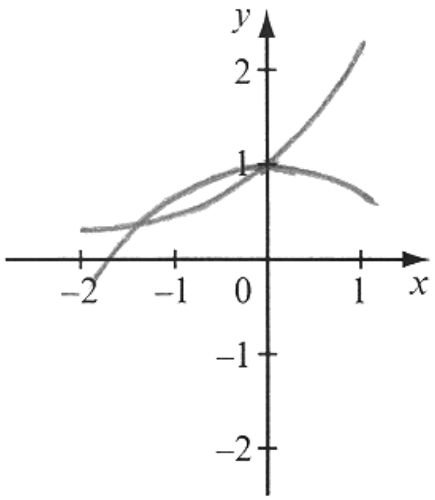
$= \frac{1}{\sqrt{10}}$ $(= 0.3162)$ ***(A1)***

$\theta = 72^\circ$ (to the nearest degree) ***(A1)*** ***(C4)***

Note: Award ***(C2)*** for a radian answer between 1.2 and 1.25.

[4 marks]

11. (a)



(A1)(A1) ***(C1)(C1)***

(b) $x = -1.29$ ***(A2)*** ***(C2)***

[4 marks]

12. $\sqrt{3-2x}=5$ ***(M1)***

$3-2x=25$ ***(A1)***

$-2x=22$ ***(A1)***

$x=-11$ ***(A1)*** ***(C4)***

OR

Let $y=\sqrt{3-2x}$

$\Rightarrow y^2=3-2x$ ***(M1)***

$\Rightarrow \quad x=\dfrac{3-y^2}{2}$ ***(A1)***

$\Rightarrow f^{-1}(x)=\dfrac{3-x^2}{2}$

$\Rightarrow f^{-1}(5)=\dfrac{3-25}{2}$ ***(M1)***

$=-11$ ***(A1)*** ***(C4)***

[4 marks]

13. (a) $3\sin^2 x+4\cos x=3(1-\cos^2 x)+4\cos x$

$=3-3\cos^2 x+4\cos x$ ***(A1)*** ***(C1)***

(b) $3\sin^2 x+4\cos x-4=0 \Rightarrow 3-3\cos^2 x+4\cos x-4=0$

$\Rightarrow 3\cos^2 x-4\cos x+1=0$ ***(A1)***

$(3\cos x-1)(\cos x-1)=0$

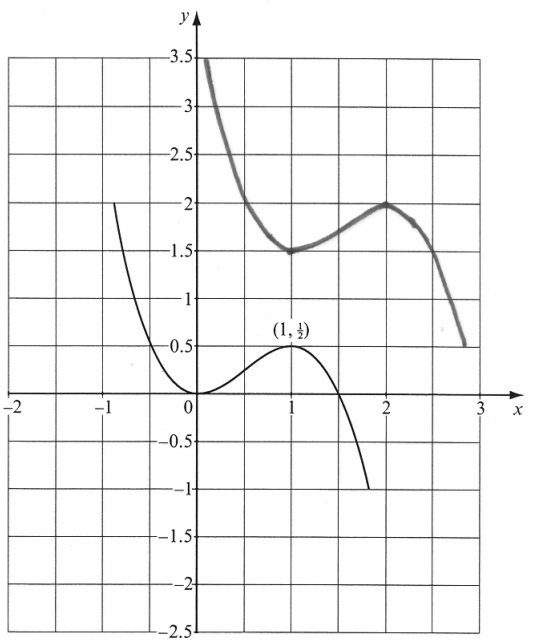
$\cos x=\dfrac{1}{3}$ or $\cos x=1$

$x=70.5^\circ$ or $x=0^\circ$ ***(A1)(A1)*** ***(C3)***

Note: Award ***(C1)*** for each correct radian answer, i.e. $x=1.23$ or $x=0$.

[4 marks]

14. (a)



(A2) ***(C2)***

(b) Minimum: $\left(1, \frac{3}{2}\right)$ ***(A1)*** ***(C1)***

Maximum: $(2, 2)$ ***(A1)*** ***(C1)***

[4 marks]

15. $\hat{OTA} = 90^\circ$ ***(A1)***

$AT = \sqrt{12^2 - 6^2}$

$= 6\sqrt{3}$

$\hat{TOA} = 60^\circ = \frac{\pi}{3}$ ***(A1)***

Area = area of triangle − area of sector

$= \frac{1}{2} \times 6 \times 6\sqrt{3} - \frac{1}{2} \times 6 \times 6 \times \frac{\pi}{3}$ ***(M1)***

$= 12.3 \text{ cm}^2 \left(\text{or } 18\sqrt{3} - 6\pi\right)$ ***(A1)*** ***(C4)***

OR

$\hat{TOA} = 60^\circ$ ***(A1)***

Area of $\Delta = \frac{1}{2} \times 6 \times 12 \times \sin 60$ ***(A1)***

Area of sector $= \frac{1}{2} \times 6 \times 6 \times \frac{\pi}{3}$ ***(A1)***

Shaded area $= 18\sqrt{3} - 6\pi = 12.3 \text{cm}^2 \left(3 \text{s.f.}\right)$ ***(A1)*** ***(C4)***

[4 marks]